Power and Interference Control with Relaying in Cooperative Cognitive Radio Networks

Weng Chon Ao, Shin-Ming Cheng, and Kwang-Cheng Chen
Dept. of Electrical Engineering National Taiwan University, Taipei 106, Taiwan
Email: r97942044@ntu.edu.tw, {smcheng, chenkc}@cc.ee.ntu.edu.tw

***Abstract*—It is well known that secondary users can take advantages of spectrum opportunities from primary users via cognitive radio technology. However, general sense cognitive radio networks allow all users to cooperatively relay the packets, especially primary users are aware of existence and activity of secondary users. Secondary users may act as cooperative relays for primary traffic. We propose a relay selection criteria such that cooperation is beneficial to both primary and secondary systems. With the suitably chosen relays, power consumption in primary transmissions is reduced and simultaneously interference from primary traffic decreases exposing spectrum opportunities for secondary traffic. We characterize the performance improvement of secondary network due to the extra spectrum opportunities by analyzing the extension of its connectivity region through percolation theory. Such self-motivated cooperation leads the way toward self-organizing wireless networks.**

I. Introduction

Facing the increasing demands of wireless services and the underutilization of licensed spectrum, cognitive radio (CR) identifying and exploiting spectrum opportunities of primary (licensed) systems has emerged as a technology to enhance spectrum utilization. The cognitive principle states that secondary users (SUs/CRs) are aware of avoiding interference with primary users (PUs), in other words, SUs are transparent to PUs.

However, general sense cognitive radio networks (CRN) allow all users to cooperatively relay the packets. When PUs are aware of the existence and activity of SUs, networking SUs and PUs forming a cooperative CRN can use a cooperative relay to greatly enhance network efficiency given a fixed bandwidth [1]. In [2], an utility-based game-theoretic model has been developed to provide incentive for cooperation between PUs and SUs. Spatial diversity gain of a *single communication link* offered by beam-forming at a cognitive/cooperative relay was discussed in [3]–[6], nevertheless, the relay may cause additional interference with other network nodes. Little work has been done to explore the problem in a *network* perspective.

We propose a secondary cooperative relay selection criteria such that cooperation is beneficial to both primary and secondary systems in a network perspective. The term "cooperative relaying" stands for "packet forwarding" that is followed in this paper. With the appropriately selected SUs acting as cooperative relays to facilitate packet transmissions between PUs, power consumption of the PUs is reduced. At the same time, interference caused by primary transmissions is also reduced exposing extra spectrum opportunities for secondary transmissions. The performance improvement of secondary system due to the extra spectrum opportunities is characterized by the extension of its connectivity region through percolation theory [7]. Our analysis shows that connectivity of the secondary network is able to be sustained with a higher PU density under the self-motivated cooperation.

The remainder of this paper is organized as follows. Section II presents the interference model with direct transmission. In Section III, we present the power and interference control model with cooperative relaying and a secondary cooperative relay selection region is described. Simulation results are provided in Section IV. In Section V, we conclude this paper.

II. Modeling Direct Transmission

A. Network Geometry

We consider a planar area where an ad hoc network of PUs and an ad hoc network of SUs coexist. Considering the randomness of distribution of nodes in a network, we assume that the spatial distribution of active primary transmitters (PTs) follows a Poisson point process with density λ_{PT}. Each PT has a dedicated primary receiver (PR) at a fixed distance r_c^{PT} away with an arbitrary direction. The spatial distribution of SUs is assumed to follow a Poisson point process with density λ_{SU} which is independent of PTs'. An SU is a potential transmitter and receiver and r_c^{SU} is denoted as its maximum transmission range. The notation $\mathcal{B}(x; r)$ used in the paper represents the circle of radius r centered at x.

B. Interference Model

To address on the coexistence between PUs and SUs, we only focus on *inter-system interference* between PUs and SUs and suitable scheduling among conflicting transmissions within their own systems is implied.

Definition 1. *Interference protection region of a PR is defined as the region $\mathcal{B}(PR; r_I^{PR})$, where no active secondary transmitter (ST) is allowed.*

For a successful reception at a PR, it satisfies

$$\frac{P_{PT}(r_c^{PT})^{-\delta}}{P_N + P_I^{SU}} \geq \eta_{PR}, \tag{1}$$

where P_{PT} is the transmitted power of a PT and is assumed to be fixed for all PTs, η_{PR} is the SINR threshold at a PR, δ is the path loss exponent, P_N is the power of background noise and P_I^{SU} is the interference power from surrounding SUs. In

978-1-4244-6826-3/10/$26.00 ©2010 IEEE

the absence of SUs (i.e., $P_I^{SU} = 0$), the received SINR level at the PR (denoted as $\overline{\eta_{PR}}$) must be larger than the SINR threshold for the additional toleration of interference. That is,

$$\overline{\eta_{PR}} \equiv \frac{P_{PT}(r_c^{PT})^{-\delta}}{P_N} > \eta_{PR}. \tag{2}$$

Under the protocol model [8], a single transmitting SU should not cause outage reception at the PR. The radius r_I^{PR} of the interference protection range of the PR can be evaluated as

$$r_I^{PR} = \left[\left(\frac{P_{PT}(r_c^{PT})^{-\delta}}{\eta_{PR}} - P_N\right)\frac{1}{P_{ST}}\right]^{-\delta^{-1}}, \tag{3}$$

which is obtained by setting $P_I^{SU} = P_{ST}(r_I^{PR})^{-\delta}$ in (1) with equality, where P_{ST} is the transmitted power of an SU.

Definition 2. *Interference region of a PT is defined as the region $\mathcal{B}(PT; r_I^{SR})$, where secondary receiver (SR) fails to receive the intended signal while maintaining the outage constraint.*

Different from the fixed distance between a pair of PT and PR (i.e., r_c^{PT}), the distance between a pair of SUs within maximum transmission range r_c^{SU} of each other is random.

Lemma 1. *The p.d.f. of the distance (denoted as R) between an SU and its neighbors within $\mathcal{B}(SU; r_c^{SU})$ (conditional on the SU has at least one neighbor within $\mathcal{B}(SU; r_c^{SU})$) can be evaluated as*

$$f_R(r) = \frac{2r}{(r_c^{SU})^2},\ 0 \le r \le r_c^{SU}. \tag{4}$$

Proof: Firstly, the maximum transmission range of an SU is obtained when no interference occurs (from PTs) and the SINR threshold at the receiver side is tight. r_c^{SU} satisfies

$$\frac{P_{ST}(r_c^{SU})^{-\delta}}{P_N} = \eta_{SR}, \tag{5}$$

where η_{SR} is the SINR threshold of an SR.

Secondly, since the spatial distribution of SUs follows a Poisson point process, given that there are neighbors of the SU within $\mathcal{B}(SU; r_c^{SU})$, the neighbors are uniformly distributed in $\mathcal{B}(SU; r_c^{SU})$. We thus have (4). ■

With an ϵ_{SR} outage probability constraint at the SR, we have

$$\mathbb{P}\left[\frac{P_{ST}R^{-\delta}}{P_N + P_I^{PT}} \ge \eta_{SR}\right] \ge 1 - \epsilon_{SR}, \tag{6}$$

where P_I^{PT} is the interference power from surrounding PTs. Under the protocol model, r_I^{SR} satisfies

$$\mathbb{P}\left[\frac{P_{ST}R^{-\delta}}{P_N + P_{PT}(r_I^{SR})^{-\delta}} \ge \eta_{SR}\right] = 1 - \epsilon_{SR}, \tag{7}$$

which is obtained by setting $P_I^{PT} = P_{PT}(r_I^{SR})^{-\delta}$ in (6) with equality. With (4) and (7), r_I^{SR} can be evaluated as

$$r_I^{SR} = \left[\frac{P_N}{P_{PT}}\left(\sqrt{1-\epsilon_{SR}}^{-\delta} - 1\right)\right]^{-\delta^{-1}}. \tag{8}$$

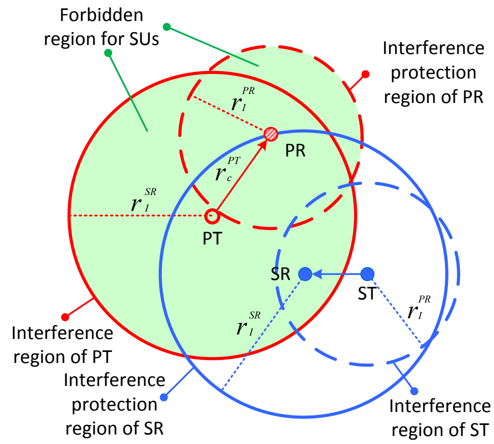

Fig. 1. Interference model

For the ease of discussion, we parameterize (3) as

$$r_I^{PR} = \beta r_c^{SU},\ \beta = \left[\left(\frac{\overline{\eta_{PR}}}{\eta_{PR}} - 1\right)\frac{1}{\eta_{SR}}\right]^{-\delta^{-1}}, \tag{9}$$

and (8) as

$$r_I^{SR} = \alpha r_c^{PT},\ \alpha = \left(\frac{\sqrt{1-\epsilon_{SR}}^{-\delta} - 1}{\overline{\eta_{PR}}}\right)^{-\delta^{-1}}. \tag{10}$$

Remark. *When the link between a pair of PT and PR is active, all STs located within distance r_I^{PR} away from the PR should be deactivated. r_I^{PR} is the radius of the interference protection region of the PR and is synonymous with the interference region of an ST. Furthermore, all SRs located within distance r_I^{SR} away from the PT cannot receive the intended signal meeting the outage constraint. r_I^{SR} is the radius of the interference region of the PT and is synonymous with the interference protection region of an SR. The above configuration is shown in Fig. 1. We assume that $r_I^{SR} > r_I^{PR}$ in the following discussion.*

To avoid causing harmful interference to primary transmissions, SUs should sense existence and activity of PUs. As illustrated in Fig. 1, we define the forbidden region for SUs of a primary link between a pair of PT and PR as the combination of the interference region of the PT and the interference protection region of the PR, and we have the lemma.

Lemma 2. *There is a (bidirectional) link between SUs A and B for opportunistic transmissions when both A and B are not located in the forbidden region of any active primary link and the distance between A and B is at most r_c^{SU}.*

C. Networking Model

While considering the bidirectional connectivity of the secondary network, SUs under the constraint suggested by **Lemma 2** can construct opportunistic links. The links establish a graph $\mathcal{G}_{SU}$ corresponding to the secondary network. The connectivity of $\mathcal{G}_{SU}$ can be defined in the percolation sense:

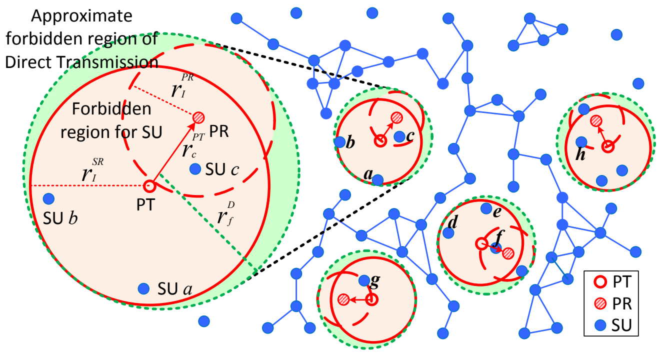

Fig. 2. A demonstration of the CRN with forbidden regions of direct primary transmissions

Definition 3. *The secondary network $\mathcal{G}_{SU}$ is connected if it percolates and a giant connected component exists.*

To analyze the connectivity of secondary network overlaid with primary network, we introduce the concept of connectivity region of a secondary network [9] as follows.

Definition 4. *The connectivity region of $\mathcal{G}_{SU}$, $\mathcal{C}_{SU}$, is defined as the set of density pairs $(\lambda_{SU}, \lambda_{PT})$ under which connectivity of the secondary system is maintained. That is,*

$$\mathcal{C}_{SU} \triangleq \{(\lambda_{SU}, \lambda_{PT}) : \mathcal{G}_{SU} \text{ is connected.}\} \quad (11)$$

The connectivity region is bounded by the critical PT density, which is defined as

Definition 5. *The critical density of the primary transmitters $\overline{\lambda_{PT}}$ is the outer bound of $\mathcal{C}_{SU}$ to ensure connectivity of the secondary network, that is,*

$$\overline{\lambda_{PT}} \triangleq \sup\{\lambda_{PT} : \exists \lambda_{SU} < \infty \text{ s.t. } \mathcal{G}_{SU} \text{ is connected.}\} \quad (12)$$

When $\lambda_{PT} > \overline{\lambda_{PT}}$, the secondary network breaks down into many finite isolated components no matter how large λ_{SU} is, which implies that spectrum opportunities are not enough for any secondary network to be connected.

D. Analysis of the Connectivity Region

To analyze the connectivity region $\mathcal{C}_{SU}$ of secondary network in large-scale random wireless network, we approximate the forbidden region for SUs of a primary direct transmission link as follows.

Definition 6. *The approximate forbidden region is defined as a circular region with radius r_f^D as shown in Fig. 2, where*

$$r_f^D = \frac{r_I^{SR} + \max\{r_I^{SR}, r_I^{PR} + r_c^{PT}\}}{2}. \quad (13)$$

Obviously, the radius of the approximate forbidden region r_f^D is monotonic with respect to r_I^{SR}, r_I^{PR} and r_c^{PT}. Moreover, it affects the connectivity of secondary network. As shown in Fig. 2, SUs a, b and c who locate in the approximate forbidden region should be deactivated to avoid causing unacceptable interference to surrounding PRs or being interfered by surrounding PTs. Thus $\mathcal{G}_{SU}$ does not include these deactivated SUs. We establish the following proposition to characterize the critical PT density of the connectivity region of the secondary network.

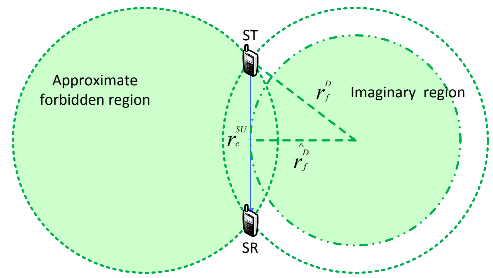

Fig. 3. Imaginary region

Proposition 1. *Given r_c^{SU}, r_c^{PT}, r_I^{PR}, and r_I^{SR}, the critical PT density $\overline{\lambda_{PT}}$ of $\mathcal{C}_{SU}$ is characterized by*

$$\overline{\lambda_{PT}} = \frac{\lambda_c(1)}{(r_f^D)^2 - \left(\frac{r_c^{SU}}{2}\right)^2}, \quad (14)$$

where $\lambda_c(1)$ is the critical density for a homogeneous network with a two-unit transmission range.

Proof: We define an imaginary region as shown in Fig. 3. The relationship between the radius $\hat{r_f^D}$ of the imaginary region and the radius r_f^D of the approximate forbidden region is

$$\hat{r_f^D} = \sqrt{(r_f^D)^2 - \left(\frac{r_c^{SU}}{2}\right)^2}. \quad (15)$$

When the imaginary regions percolate, the approximate forbidden regions percolate and overlap with each other with "width" r_c^{SU} corresponding to the maximum transmission range of SUs. It indicates that no transmission is able to occur between SUs in different isolated clusters. The necessary condition of existence of a giant connected component in the secondary system is that the imaginary regions do not percolate. Continuum percolation theory [9], [10] shows that percolation of the imaginary regions occurs at $\lambda_c(1)/(\hat{r_f^D})^2$, in other words,

$$\overline{\lambda_{PT}} = \lambda_c(1)/(\hat{r_f^D})^2. \quad (16)$$

Applying (15) into (16), we have (14). ■

From **Proposition 1**, we have the following corollary

Corollary 1. *When r_f^D decreases, $\overline{\lambda_{PT}}$ increases and the connectivity region $\mathcal{C}_{SU}$ of the secondary system is extended.*

III. Modeling Cooperative Relay

When direct packet transmissions occur between pairs of PT and PR, all SUs within the forbidden regions are deactivated. With the assumption that PUs are aware of the existence of SUs, this section discusses the benefits for both primary and secondary systems by suitably choosing deactivated SUs as cooperative relays for primary transmissions. We focus on using a single deactivated SU as a relay help forwarding

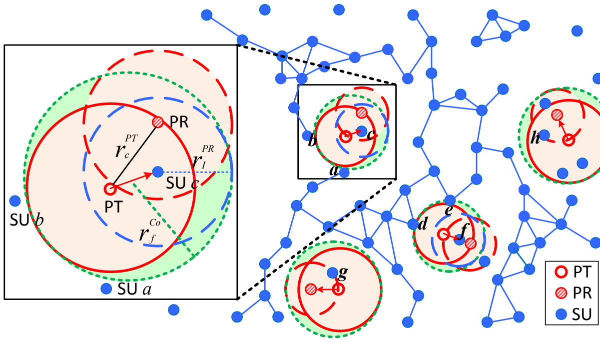

Fig. 4. A demonstration of cooperative relaying in the CRN

packets between a pair of PT and PR. After cooperation, the one-hop transmission between a pair of PT and PR becomes a two-hop transmission. The first hop is between the PT and the SU relay (denoted as SRe) while the second hop is between the SRe and the PR.

Fig. 4 shows the snapshot of the first hop transmission in cooperative relaying where SUs c and f are chosen as SRes and SUs a, b, d, and e are no longer located in the reduced forbidden region and can be activated. The intuition of importance of choosing an appropriate relay among deactivated SUs is that it dominates the power consumption of primary transmissions and connectivity of secondary network. Please realize that although the forbidden region shrinks after cooperation, its density doubles (two-hop). In the following discussion, we define the "beneficial" region from where a deactivated SU may be chosen acting as a cooperative relay.

From the perspective of PUs,

Lemma 3. *To reduce the power consumption of primary transmissions, the SRe should be located within the region defined by,*

$$d^{\delta}_{PT\rightarrow SRe} + d^{\delta}_{SRe\rightarrow PR} \leq (r_c^{PT})^{\delta} \tag{17}$$

where $d_{PT\rightarrow SRe}$ is the distance between the PT and the SRe, $d_{SRe\rightarrow PR}$ is the distance between the SRe and the PR, and δ is the path loss exponent with value larger than 2 in general.

Proof: Obviously, (17) ensures the total power consumption of the two-hop transmission is less than that of direct transmission. ∎

From the perspective of SUs, cooperative relaying by SRe decreases the radius of the corresponding forbidden region. The relation between $d_{PT\rightarrow SRe}$ and the radius r_f^{Co} of the forbidden region in cooperative relaying is presented in the following lemma.

Lemma 4. *The radius r_f^{Co} of the approximate forbidden region of the first hop transmission with distance $d_{PT\rightarrow SRe}$ in cooperative relaying is*

$$r_f^{Co} = \frac{\alpha d_{PT\rightarrow SRe} + \max\{\alpha d_{PT\rightarrow SRe}, r_I^{PR} + d_{PT\rightarrow SRe}\}}{2}$$
$$= \begin{cases} \alpha d_{PT\rightarrow SRe}, & \text{if } r_I^{PR} < (\alpha-1) d_{PT\rightarrow SRe} \\ \frac{(1+\alpha)d_{PT\rightarrow SRe} + r_I^{PR}}{2}, & \text{if } (\alpha-1) d_{PT\rightarrow SRe} \leq r_I^{PR}. \end{cases} \tag{18}$$

Proof: When cooperative relaying is adopted, the PT adjusts its transmission power P_{PT} according to the distance between the SRe, $d_{PT\rightarrow SRe}$, while keeping $\overline{\eta_{PR}}$ fixed. From (10), the radius of interference region of PT in cooperative relaying becomes $\alpha d_{PT\rightarrow SRe}$. The SRe plays a role of PR in cooperative relaying and its interference protection region thus equals to that of a normal PR (i.e., with radius r_I^{PR}).

By applying updated interference region of PT and interference protection region of SRe into (13), we have (18). ∎

In the following proposition, we derive the maximum distance between a chosen SRe and the PT (and between the SRe and the PR) in cooperative relaying to ensure that the connectivity region of secondary network is at least as large as (or better than) that with direct transmission.

Proposition 2. *To ensure extension of the connectivity region of secondary network, the chosen SRe should be located within the intersection of two circles with radius r_{th}^{Co} centered at the PT and PR respectively. r_{th}^{Co} can be computed as*

$$r_{th}^{Co} = \begin{cases} \frac{1}{\alpha}\sqrt{\frac{(r_f^D)^2}{2} + \frac{(r_c^{SU})^2}{8}}, & \text{if } r_I^{PR} < (\alpha-1) r_{th}^{Co} \\ \frac{1}{1+\alpha}\left(2\sqrt{\frac{(r_f^D)^2}{2} + \frac{(r_c^{SU})^2}{8}} - r_I^{PR}\right), & \text{if } (\alpha-1) r_{th}^{Co} \leq r_I^{PR} \end{cases} \tag{19}$$

Proof: We firstly specify the parameters in cooperative relaying achieving the same performance as direct transmission, then we clarify the condition under which we can do better. Firstly, to fairly compare the two-hop transmission in cooperative relaying with the direct transmission, the density of reduced forbidden regions (primary traffic) in the two-hop transmission is defined as twice that of forbidden regions in direct transmission. Thus, **Proposition 1** indicates $2\overline{\lambda_{PT}} = \lambda_c(1)/(r_f^{\hat{C}o})^2$, where $r_f^{\hat{C}o}$ is the radius of the corresponding imaginary region of r_f^{Co}. With (15), (16) and some algebraic, we have

$$r_f^{\hat{C}o} = \sqrt{\frac{\lambda_c(1)}{2\overline{\lambda_{PT}}}} = \frac{r_f^{\hat{D}}}{\sqrt{2}} = \sqrt{\frac{(r_f^D)^2}{2} - \frac{(r_c^{SU})^2}{8}} \tag{20}$$

and

$$r_f^{Co} = \sqrt{(r_f^{\hat{C}o})^2 + \left(\frac{r_c^{SU}}{2}\right)^2} = \sqrt{\frac{(r_f^D)^2}{2} + \frac{(r_c^{SU})^2}{8}}. \tag{21}$$

Let r_{th}^{Co} denote the upper threshold of distance between the PT and the SRe (and between the SRe and the PR) in cooperative relaying achieving the same connectivity region of the secondary network as the one in direct transmission. Substituting $d_{PT\rightarrow SRe}$ with r_{th}^{Co} in (18) and combining with (21), we arrive (19), where r_f^D is defined in (13). Secondly, when we locate the SRe within the intersection of two circles with radius r_{th}^{Co} centered at the PT and PR respectively, we have $\max(d_{PT\rightarrow SRe}, d_{SRe\rightarrow PR}) \leq r_{th}^{Co}$. It leads to a decrease in r_f^{Co}, and from **Corollary 1** we have an increase in $\overline{\lambda_{PT}}$ under which the connectivity region of secondary network is extended. ∎

An example is shown in Fig. 5. Please note that for a

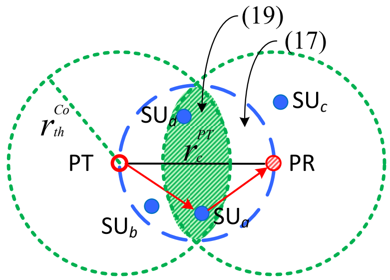

Fig. 5. An example with $\alpha = \beta = 3$, $r_c^{PT} = 100$m, $r_c^{SU} = 50$m and $r_{th}^{Co} \approx 69$m; the circular region labeled (17) is plotted with $\delta = 2$.

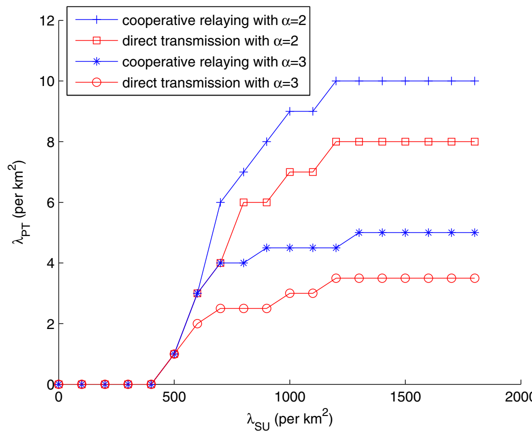

Fig. 6. Extension of the connectivity region of secondary network

feasible SRe, both (17) and (19) are needed to be satisfied. The area of intersection of the two circles is given by

$$\mathcal{A}(r_{th}^{Co}) \equiv 2(r_{th}^{Co})^2 \cos^{-1}\left(\frac{r_c^{PT}}{2r_{th}^{Co}}\right) - r_c^{PT}\sqrt{(r_{th}^{Co})^2 - \left(\frac{r_c^{PT}}{2}\right)^2} \tag{22}$$

The probability of having at least one SU within $\mathcal{A}(r_{th}^{Co})$ is $1 - e^{-\lambda_{SU}\mathcal{A}(r_{th}^{Co})}$ which tends to 1 when λ_{SU} is large enough.

IV. Simulation Results

Fig. 6 shows that the connectivity region of secondary network extends in cooperative relaying compared with the one in direct transmission. The network size in simulation is $3\text{km} \times 3\text{km}$ with $r_c^{PT} = 100$m, $r_c^{SU} = 50$m, $\alpha = \beta = 2$ and 3. The secondary network is considered to be connected (in percolation sense) when there exists at least one L-R crossing [9] [10]. As can be seen in the figure, there exists a threshold around $\lambda_{SU} = 500$ (per km^2) corresponding to the critical SU density for percolation in the secondary network. When λ_{SU} increases, all curves increase, stabilize and bounded above by the critical PT density indicating that the forbidden regions percolate.

Fig. 7 shows that with relaying the normalized average power consumption of primary transmissions gradually reduces and levels off. We have $r_c^{PT} = 100$m, $r_{th}^{Co} \approx 69$m and $\delta = 4$. The lower curve stabilizes since the probability of having at least one SU within $\mathcal{A}(r_{th}^{Co})$ tends to one when λ_{SU} increases.

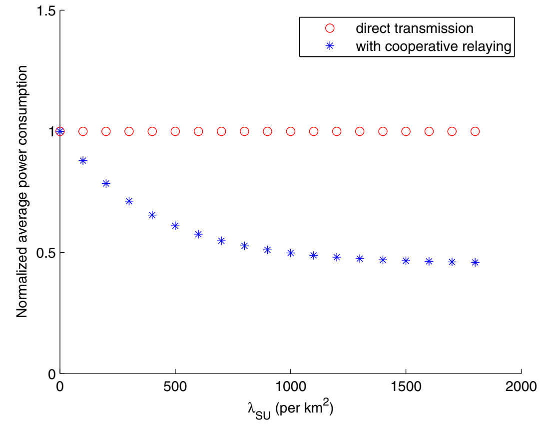

Fig. 7. Power reduction of primary transmission

V. Conclusion

We have studied the problem of networking appropriately selected SUs as cooperative relays to facilitate packet transmissions between PUs, to reduce interference caused by primary transmissions exposing extra spectrum opportunities for secondary transmissions. An analytic description of the feasible relay selection region is provided and verified through simulation results. Under cooperation with the selected SU relays, power consumption is reduced for primary packet transmissions and simultaneously connectivity region of secondary network is extended. For further studies, different cooperative schemes can be applied in this framework.

References

[1] C.-H. Huang, Y.-C. Lai, and K.-C. Chen, "Network capacity of cognitive radio relay network," *Physical Communication*, vol. 1, no. 2, pp. 112–120, June 2008.

[2] J. Zhang and Q. Zhang, "Stackelberg game for utility-based cooperative cognitive radio networks," in *Proc. ACM MobiHoc '09*, May 2009, pp. 23–32.

[3] Q. Zhang, J. Jia, and J. Zhang, "Cooperative relay to improve diversity in cognitive radio networks," *IEEE Commun. Mag.*, vol. 47, no. 2, pp. 111–117, Feb. 2009.

[4] Y. Zhu and H. Zheng, "Understanding the impact of interference on collaborative relays," *IEEE Trans. Mobile Comput.*, vol. 7, no. 6, pp. 724–735, June 2008.

[5] C. Sun and K. B. Letaief, "User cooperation in heterogeneous cognitive radio networks with interference reduction," in *Proc. IEEE ICC '08*, May 2008, pp. 3193–3197.

[6] O. Simeone, Y. Bar-Ness, and U. Spagnolini, "Stable throughput of cognitive radios with and without relaying capability," *IEEE Trans. Commun.*, vol. 55, no. 12, pp. 2351–2360, Dec. 2007.

[7] W. Ren, Q. Zhao, and A. Swami, "Connectivity of cognitive radio networks: proximity vs. opportunity," in *Proc. ACM CoRoNet '09*, Sept. 2009, pp. 37–42.

[8] Y. Shi and Y. Hou, "Optimal power control for multi-hop software defined radio networks," in *Proc. IEEE Infocom '07*, May 2007, pp. 1694–1702.

[9] W. Ren, Q. Zhao, and A. Swami, "Connectivity of heterogeneous wireless networks," *IEEE Trans. Inf. Theory*, 2009, submitted for publication. [Online]. Available: http://arxiv.org/abs/0903.1684

[10] R. Meester and R. Roy, *Continuum Percolation*. Cambridge Univerisity Press, New York, 1996.